RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4721

Core Mathematics 1

Monday **10 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
- Answer booklet
- Graph paper
- List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This question paper consists of 3 printed pages and 1 blank page.

1 (i) Express 11^{-2} as a fraction. [1]

(ii) Evaluate $100^{\frac{3}{2}}$. [2]

(iii) Express $\sqrt{50} + \frac{6}{\sqrt{3}}$ in the form $a\sqrt{2} + b\sqrt{3}$, where a and b are integers. [3]

2 Given that $2x^2 - 12x + p = q(x - r)^2 + 10$ for all values of x, find the constants p, q and r. [4]

3 (i) The curve $y = 5\sqrt{x}$ is transformed by a stretch, scale factor $\frac{1}{2}$, parallel to the x-axis. Find the equation of the curve after it has been transformed. [2]

(ii) Describe the single transformation which transforms the curve $y = 5\sqrt{x}$ to the curve $y = (5\sqrt{x}) - 3$. [2]

4 Solve the simultaneous equations

$$x^2 - 3y + 11 = 0, \qquad 2x - y + 1 = 0.$$ [5]

5 On separate diagrams,

(i) sketch the curve $y = \frac{1}{x}$, [2]

(ii) sketch the curve $y = x(x^2 - 1)$, stating the coordinates of the points where it crosses the x-axis, [3]

(iii) sketch the curve $y = -\sqrt{x}$. [2]

6 (i) Calculate the discriminant of $-2x^2 + 7x + 3$ and hence state the number of real roots of the equation $-2x^2 + 7x + 3 = 0$. [3]

(ii) The quadratic equation $2x^2 + (p + 1)x + 8 = 0$ has equal roots. Find the possible values of p. [4]

7 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \frac{1}{2}x^4 - 3x$, [2]

(ii) $y = (2x^2 + 3)(x + 1)$, [4]

(iii) $y = \sqrt[5]{x}$. [3]

8 The length of a rectangular children's playground is 10 m more than its width. The width of the playground is x metres.

(i) The perimeter of the playground is greater than 64 m. Write down a linear inequality in x. [1]

(ii) The area of the playground is less than 299 m^2. Show that $(x - 13)(x + 23) < 0$. [2]

(iii) By solving the inequalities in parts **(i)** and **(ii)**, determine the set of possible values of x. [5]

9 **(i)** Find the gradient of the curve $y = 2x^2$ at the point where $x = 3$. [2]

(ii) At a point A on the curve $y = 2x^2$, the gradient of the normal is $\frac{1}{8}$. Find the coordinates of A. [3]

Points $P_1\ (1, y_1)$, $P_2\ (1.01, y_2)$ and $P_3\ (1.1, y_3)$ lie on the curve $y = kx^2$. The gradient of the chord P_1P_3 is 6.3 and the gradient of the chord P_1P_2 is 6.03.

(iii) What do these results suggest about the gradient of the tangent to the curve $y = kx^2$ at P_1? [1]

(iv) Deduce the value of k. [3]

10 The points D, E and F have coordinates $(-2, 0)$, $(0, -1)$ and $(2, 3)$ respectively.

(i) Calculate the gradient of DE. [1]

(ii) Find the equation of the line through F, parallel to DE, giving your answer in the form $ax + by + c = 0$. [3]

(iii) By calculating the gradient of EF, show that DEF is a right-angled triangle. [2]

(iv) Calculate the length of DF. [2]

(v) Use the results of parts **(iii)** and **(iv)** to show that the circle which passes through D, E and F has equation $x^2 + y^2 - 3y - 4 = 0$. [5]

1 (i)	$11^{-2} = \frac{1}{121}$	B1 1	$\frac{1}{121}$ ($\frac{1}{11^2}$ = B0)
(ii)	$100^{\frac{3}{2}} = 1000$	M1 A1 2	Square rooting or cubing soi 1000
(iii)	$\sqrt{50} + \frac{6}{\sqrt{3}}$	B1	$5\sqrt{2}$ (allow $\pm$)
	$= 5\sqrt{2} + \frac{6\sqrt{3}}{3}$	M1	Attempt to rationalise $\frac{6}{\sqrt{3}}$
	$= 5\sqrt{2} + 2\sqrt{3}$	A1 3	cao
		<u>6</u>	
2	*q*=2	B1	(allow embedded values)
	r=3	B1	
		M1	$qr^2 + 10 = p$ or other correct method
	p=28	A1√ 4	
		<u>4</u>	
3(i)	$y = 5\sqrt{2x}$	M1	$\sqrt{2x}$ or $\sqrt{\frac{x}{2}}$ seen
		A1 2	$y = 5\sqrt{2x}$
(ii)	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	B1	Translation
		B1 2	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ o.e.
		<u>4</u>	

4	Either $y = 2x + 1$ or $y = \frac{x^2 + 11}{3}$	M1	Substitute for x/y or attempt to get an equation in 1 variable only
	$x^2 - 6x + 8 = 0$	A1	Obtain correct 3 term quadratic
	$(x-2)(x-4) = 0$	M1	Correct method to solve 3 term quadratic
	$x = 2 \quad x = 4$	A1	<u>or</u> one correct pair of values B1
	$y = 5 \quad y = 9$	A1	second correct pair of values B1 c.a.o
	OR $x = \frac{y-1}{2}$ $\frac{(y-1)^2}{4} - 3y + 11 = 0$ $y^2 - 14y + 45 = 0$ $(y-5)(y-9) = 0$ $y = 5 \quad y = 9$ $x = 2 \quad x = 4$		
			<u>SR</u> If solution by graphical methods: setting out to draw a parabola <u>and</u> a line M1 both correct A1 reading off of coordinates at intersection point(s) M1 one correct pair A1 second correct pair A1 OR No working shown: one correct pair B1 second correct pair B1 full justification that these are the only solutions B3
		<u>5</u>	

5 (i)		B1	Correct curve in +ve quadrant
		B1 2	in –ve quadrant
(ii)		M1	Positive cubic with clearly seen max and min points
		A1	(-1,0) (0,0) (1,0) Any one point stated or marked on sketch
	(-1,0) (0,0) (1,0)	A1 3	Curve passes through all 3 points and no extras stated or marked on sketch
(iii)		B1	Graph only in bottom right hand quadrant
		B1 2	Correct graph, passing through origin
		7	

6 (i)	$49-4\times-2\times3=73$	M1	Uses b^2-4ac
	2 real roots	A1	73
		B1$\sqrt{}$ 3	2 real roots (ft from their value)
(ii)	$(p+1)^2-64=0$ or $2[(x+\frac{p+1}{4})^2-\frac{(p+1)^2}{16}+4]=0$	M1	Attempts b^2-4ac = 0 (involving p) or attempts to complete square (involving p)
		A1	$(p+1)^2-64=0$ aef
	p= -9,7	B1	*p*= -9
		B1 4	p= 7
		7	

7 (i)	$\frac{dy}{dx}=2x^3-3$	B1 B1 2	1 term correct Completely correct (+c is an error, but only penalise once)
(ii)	$y=2x^3+2x^2+3x+3$ $\frac{dy}{dx}=6x^2+4x+3$	M1 A1 A1 A1 4	Attempt to expand brackets $2x^3+2x^2+3x+3$ 2 terms correct Completely correct <u>SR</u> Recognisable attempt at product rule M1 one part correct A1 second part correct A1 final simplified answer A1
(iii)	$y=x^{\frac{1}{5}}$ $\frac{dy}{dx}=\frac{1}{5}x^{-\frac{4}{5}}$	B1 B1 B1 3 <u>9</u>	$x^{\frac{1}{5}}$ soi $\frac{1}{5}x^c$ $kx^{-\frac{4}{5}}$
8(i)	$2[10+x+x]>64$	B1 1	$20+4x>64$ o.e.
(ii)	$x(x+10)<299$ $x^2+10x-299<0$ $(x-13)(x+23)<0$	B1 B1 2	$x(x+10)<299$ Correctly shows $(x-13)(x+23)<0$ **AG** <u>SR</u> <u>Complete</u> proof worked backward B2
(iii)	$x>11$ $(x-13)(x+23)<0$ $-23<x<13$ $\therefore 11<x<13$	B1√ M2 A1 B1 5 <u>8</u>	$x>11$ ft from their (i) Correct method to solve $(x-13)(x+23)<0$ eg graph $-23<x<13$ seen in this form or as number line <u>SR</u> if seen with no working B1

9(i)	$\frac{dy}{dx} = 4x$ At x=3 , $\frac{dy}{dx} = 12$	B1 B1 2	$4x$ 12
(ii)	Gradient of tangent = - 8 $4x = -8$ $x = -2$ $y = 8$	M1 A1 A1 3	$\frac{dy}{dx} = -8$ x=-2 y=8
(iii)	Gradient = 6	B1 1	Gradient = or approaches 6
(iv)	$\frac{dy}{dx} = 2kx$ $x = 1$ $\frac{dy}{dx} = 2k$ $k = 3$	M1 M1 A1 $\sqrt{}$ 3 9	$\frac{dy}{dx} = 2kx$ $\frac{dy}{dx} = 2k$ $k = 3$ CWO

10(i)	Gradient DE = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$ (any working seen must be correct)
(ii)	$y-3=-\frac{1}{2}(x-2)$	M1	Correct equation for straight line, any gradient, passing through F
		A1	$y-3=-\frac{1}{2}(x-2)$ aef
	$x+2y-8=0$	A1 3	$x+2y-8=0$ (this form but can have fractional coefficients e.g. ½x + y – 4 = 0
(iii)	Gradient EF = $\frac{4}{2}$ =2	B1	Correct supporting working must be seen
	$-\frac{1}{2}\times 2=-1$	B1 2	Attempt to show that product of their gradients = - 1 o.e.
(iv)	DF=$\sqrt{4^2+3^2}=5$	M1	$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ used
		A1 2	5
(v)	DF is a diameter as angle DEF is a right angle.	B1	Justification that DF is a diameter
	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$	B1	Mid-point of DF <u>or</u> centre of circle is $(0,1\frac{1}{2})$
	Radius = 2.5	B1	Radius = 2.5
	$x^2+(y-\left(\frac{3}{2}\right)^2)=\left(\frac{5}{2}\right)^2$	B1√	$x^2+(y-\left(\frac{3}{2}\right)^2)=\left(\frac{5}{2}\right)^2$
	$x^2+y^2-3y+\frac{9}{4}=\frac{25}{4}$		
	$x^2+y^2-3y-4=0$	B1 5	$x^2+y^2-3y-4=0$ obtained correctly with at least one line of intermediate working. <u>SR</u> For working that only shows $x^2+y^2-3y-4=0$ is equation for a circle with centre $(0,1\frac{1}{2})$ B1 radius 2.5 B1
		<u>13</u>	